JOURNAL OF FINANCIAL INTERMEDIATION 1, 242–256 (1991)

The Macroeconomic Effects of Bank Runs: An Equilibrium Analysis*

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Received March 29, 1990

This paper offers a model of intermediation in the Diamond–Dybvig tradition in which both fiat currency and bank deposits are present. The behavior of the economy's price level, deposit–currency ratio, and money supply is compared across equilibria in which bank runs do and do not occur. It is shown that the behavior of these variables in the presence and absence of runs is consistent with that observed in the United States during the period from 1929 to 1933. *Journal of Economic Literature* Classification Numbers: E44, G21. © 1991 Academic Press, Inc.

1. INTRODUCTION

During the period from 1929 to 1933, more than 10,000 commercial banks in the United States failed, many of them after having experienced a run. In their analysis of the macroeconomic effects of these bank failures, Friedman and Schwartz (1963) note that while the money supply, the price level, bank deposits, and the deposit–currency ratio all declined, the stock of high-powered money remained relatively unchanged.[1] Cagan (1965) reports similar results in his study of the behavior of the U.S. money supply and its component parts.

* I thank two anonymous referees, an editor of this journal, David Papell, Frederick Joutz, and seminar participants at The George Washington University, University of Virginia, Federal Reserve Bank of Dallas, and the University of Houston for their many useful comments. As usual, none of these individuals is responsible for any remaining errors.

[1] Friedman and Schwartz define these variables as follows: money supply, the sum of currency in circulation, demand deposits, and time deposits; the price level, the wholesale price index; bank deposits, demand plus time deposits; deposit–currency ratio, the ratio of bank deposits to currency in circulation; high-powered money, currency in circulation plus reserve deposits of commercial banks held at Federal Reserve Banks.

1042-9573/91 $3.00

This paper provides an example of a general equilibrium model of intermediation whose equilibria are consistent with the Friedman–Schwartz–Cagan (henceforth FSC) observations. These results are obtained using a version of the risk-sharing model developed by Bryant (1980) and Diamond and Dybvig (1983) which includes both fiat currency and bank deposits. Furthermore, agents or intermediaries on their behalf hold both deposits and currency even though deposits dominate currency in rate of return, at least in the absence of bank runs.[2]

In order that dominated currency be held, I impose a type of reserve requirement on fiat currency. Following Wallace (1984) and Miller and Wallace (1985), I assume that the real value of each agent's or intermediary's stock of fiat currency (henceforth referred to as real balances) must be at least a given percentage of the agent's saving.[3,4] A reserve requirement of this type implies that currency will be demanded in all states of the world and that currency and deposits need not be perfect substitutes. This distinction permits an analysis of bank runs that includes both deposits and currency, a necessary condition for a model of intermediation to confront the observations that FSC report.

The form of the reserve requirement that I use is different from that found in Bryant (1980, Sect. 3). Whereas Bryant uses the term "reserve requirement" to refer to the currency that intermediaries freely choose to hold as part of their optimal portfolio, my reserve requirement is an exogenously imposed constraint. This way of modeling a reserve requirement is designed to capture one of the roles that reserve requirements play in most economies, namely to provide a strictly positive demand for a particular type of government liability. One rationale for governments' imposition of such a constraint is that it provides them with a steady (although limited) form of raising revenue.

In contrast to the approach taken here, most models of intermediation in the risk-sharing tradition do not include fiat currency, just bank deposits.[5] Consequently they are unable to confront FSC's observations. On the other hand, in models where currency is included, it typically earns the same return as do deposits, at least in the absence of bank runs.[6] Since the absence of rate-of-return dominance implies that currency and deposits are perfect substitutes, these models are also unable to confront FSC's observations.

[2] Both Hicks (1935) and Bryant and Wallace (1980) argue that rate-of-return dominance is a necessary feature of any model in which money and other assets exist simultaneously.

[3] In his discussion of "real balances" in *The New Palgrave*, Patinkin (1987) attributes to Keynes (1930, Vol. 1, p. 222) one of the earliest uses of this expression.

[4] As will be shown in footnote 9, this type of reserve requirement is equivalent to one where real balances are at least a given percentage of deposits.

[5] Examples include Diamond and Dybvig (1983), Bhattacharya and Gale (1987), Jacklin (1987), Freeman (1988), Smith (1988), and Engineer (1989).

[6] See, for example, Bryant (1980).

As in Diamond and Dybvig (1983) and Bhattacharya and Gale (1987), I assume that each agent lives for either two or three periods, and that this is private information that is not revealed to the agent until after he makes his saving decisions. I also assume, as they do, that there exists an investment technology that yields a higher return when held from the first to the third period than when held from the first to the second. Consequently, an intermediary can make all agents better off by insuring them against the possibility that they live for only two periods rather than for three.

Waldo (1985) offers an earlier example of an intermediation model designed to explain FSC's observations about the consequences of bank runs. However, while both currency and deposits are also present in his model, his approach to modeling intermediation and the demand for money is different from mine. Waldo assumes that all agents live for three periods and that intermediaries do not engage in any risk sharing. Instead, intermediaries provide a means for savers to satisfy an unknown "liquidity assessment" by providing them indirect access to asset markets that they cannot otherwise access due to their inability to meet these markets' fixed costs of entry.

Turning to the question of currency, my assumption of a reserve requirement implies that agents hold currency in a world where bank runs are possible, as well as in one where they are not. This is not the case in Waldo, however. Instead, if bank runs are a possibility, then Waldo shows that currency can dominate deposits when a bank run occurs. Since there are no fixed costs to enter the market for currency and all savers are risk averse, currency will be a part of each saver's portfolio. On the other hand, if bank runs can never occur, then agents do not hold any currency, since under Waldo's assumptions deposits always dominate currency in such a world. This result would appear to make it difficult to verify FSC's observations about the behavior of the deposit–currency ratio in the presence and absence of bank runs.[7] No such difficulty arises here since the reserve requirement guarantees that currency is demanded in both states of the world.

The next section describes the model. Section 3 first describes the nonoptimal competitive solution that occurs when no mechanism exists to insure agents against being only two-period lived. It also shows the optimal competitive solution that arises when risk sharing is possible. Section 4 considers equilibria where bank runs do and do not occur. In particular, it shows how banks can provide optimal risk sharing when bank runs do not occur. Section 5 illustrates how these equilibria are consistent with FSC's observations. The paper concludes in Section 6 with a brief summary.

[7] Indeed, in his discussion of his model's empirical implications, Waldo (1985, Sect. 5) does not mention the deposit–currency ratio.

2. THE MODEL

Consider an overlapping generations economy where at each date $t \geq 1$, a continuum of agents (taken to be of measure 1) is born. Each agent born at time t is endowed with one unit of the economy's lone consumption good during time t and no units at any other date. Following Diamond and Dybvig (1983), Bhattacharya and Gale (1987), Freeman (1988), and others, I assume that each generation is composed of two types of agents: a fraction λ $(0 < \lambda < 1)$ who live for two periods (early diers) and a fraction $1 - \lambda$ who live for three periods (late diers). Both early and late diers consume only during their final period of life. While λ is assumed to be nonstochastic and public information, an agent's type is private information that does not become revealed to the agent until his second period of life.

Let $c_i(t)$ be the time $t + i$, $i = 1, 2$, consumption of an agent born at time $t \geq 1$. Since an agent's type determines when he consumes, it also determines his utility. Thus, let each agent's utility function be given by $U[c_1(t), c_2(t), \theta] = \theta u[c_1(t)] + (1 - \theta)u[c_2(t)]$, where $\theta = 0$ if the agent is a late dier and $\theta = 1$ if the agent is an early dier. $u(\cdot)$ is increasing, strictly concave, and twice continuously differentiable and satisfies $-cu''(c)/u'(c) > 1$ everywhere.

At $t = 1$ there also exist agents of generations -1 and 0 who are taken to be continua of measures $1 - \lambda$ and 1, respectively. Since these agents are in either their second or their third period of life, their types are known to them. This implies that each such agent's quantity of consumption fully characterizes his preferences. Hence, the utility function for each member of generation -1 is given by $c_2(-1)$. Similarly, the utility function for the fraction λ of generation 0 who are early diers is given by $c_1(0)$ while that for the fraction $1 - \lambda$ who are late diers is given by $c_2(0)$.

The economy's investment technology, storage, is assumed to be available only to agents who are in their first period of life. Storage may be accessed either directly or indirectly through an intermediary. Time t storage, $s(t)$, earns a gross rate of return of $X > 1$ per period. Storage that is not consumed after one period remains in place and hence earns a two-period return of X^2.

In addition to storage, there exists a second asset, namely fiat currency. I take the supply of fiat currency to be fixed at $H > 0$. Since both Federal Reserve Notes and reserve deposits are forms of fiat currency, this assumption corresponds to FSC's observation that the stock of high-powered money was relatively stable during the early 1930s. Unlike storage, which an agent can purchase only when he is young, currency can be purchased during any period of an agent's life. This assumption represents the idea that currency is more "liquid" than is storage and therefore can be acquired more readily.

Recall that at $t = 1$ the measure of agents of generations -1 and 0 equals $2 - \lambda$.[8] Assuming an equal per capita initial allocation of currency, it follows that at $t = 1$ the members of generation -1 hold $(1 - \lambda)H/(2 - \lambda)$ units of currency while the members of generation 0 hold the remaining $H/(2 - \lambda)$ units.

The reserve requirement takes the following form: Let $p(t)$ be the time t value of currency in terms of time t good (the inverse of the time t price level) and $h(t)$ be the stock of currency that a young agent acquires at time t. Then each young agent must allocate at least the amount $\beta \in (0, 1)$ of his endowment to real balances, $q(t) = p(t)h(t)$.[9] Since the reserve requirement guarantees that currency is held every period, it follows that $p(t) > 0$ for all $t \geq 1$. Therefore, both $R_1(t) = p(t + 1)/p(t)$, the one-period return on time t real balances, and $R_2(t) = p(t + 2)/p(t)$, the two-period return, are well defined for all $t \geq 1$. Whereas the return to storage is exogenous, the returns to real balances are endogenous. In particular, they will depend upon whether a bank run occurs or not.[10]

3. EQUILIBRIA IN THE ABSENCE OF AN INTERMEDIARY

Consider a competitive economy in which agents hold assets directly. Since all agents born at time $t \geq 1$ make their saving decisions before they ascertain their type, each such agent maximizes expected utility, $E(U[c_1(t), c_2(t), \theta])$, conditional upon his knowledge of λ. By the law of large numbers, λ is the probability of being an early dier. Hence, each agent born at time $t \geq 1$ solves the following problem: choose $c_1(t)$, $c_2(t)$, $q(t)$, and $s(t)$ to maximize $\lambda u[c_1(t)] + (1 - \lambda)u[c_2(t)]$ subject to

$$c_1(t) = R_1(t)q(t) + Xs(t), \tag{1a}$$

$$c_2(t) = R_2(t)q(t) + X^2s(t), \tag{1b}$$

$$q(t) + s(t) = 1, \tag{1c}$$

$$q(t) \geq \beta. \tag{1d}$$

After substituting Eqs. (1a)–(1c) into the utility function, the first-order conditions for this problem can be written as

[8] More generally, this is true of generations $t - 2$ and $t - 1$ for all $t \geq 1$.

[9] Since each agent's unit of endowment is allocated between real balances and storage, the reserve requirement can also be expressed as $q(t) \geq \gamma s(t)$, where $\gamma = \beta/(1 - \beta)$. This interpretation is especially useful once banks are introduced since it implies that they hold a given percentage of deposits (i.e., storage) as real balances. Note that γ can exceed one because deposits do not represent each agent's total saving.

[10] This is in contrast to Waldo (1985) in which the return to currency is exogenous.

$$\lambda(X - R_1(t))u_1' + (1 - \lambda)(X^2 - R_2(t))u_2' - \mu \leq 0; \qquad = 0 \text{ if } s(t) > 0, \tag{2a}$$

$$\mu(1 - s(t) - \beta) = 0, \tag{2b}$$

where $u_1' = u'(c_1(t))$, $u_2' = u'(c_2(t))$, and $\mu \geq 0$ is a Lagrange multiplier for the reserve requirement, Eq. (1d). The solution to this problem determines a pair of asset demand functions, each of which is a function of λ and relative rates of return.

Turning to the members of generations -1 and 0, because their type is known to them, each agent maximizes his consumption by inelastically supplying all of his currency during the period in which he consumes. For the members of generation -1 and the early diers of generation 0, this is $t = 1$; for the late diers of generation 0, this is $t = 2$. Furthermore, due to the economy's stationarity, at all dates $t \geq 2$, the same is true of the earlier diers of generation $t - 1$ and the late diers of generation $t - 2$. Hence, at each date $t \geq 1$, the supply of fiat currency equals $H/(2 - \lambda)$ where the early diers of generation $t - 1$ supply $\lambda H/(2 - \lambda)$ and the late diers of generation $t - 2$ supply $(1 - \lambda)H/(2 - \lambda)$.[11]

In a competitive equilibrium, at each date $t \geq 1$, $R_1(t)$ and $R_2(t)$ adjust to clear the market for real balances. Therefore, for all $t \geq 1$, $R_1(t)$ and $R_2(t)$ must satisfy

$$q(t) = p(t)H/(2 - \lambda). \tag{3}$$

With both the supply of fiat currency and the population constant, Eq. (3) implies that $R_1(t) = R_2(t) = 1$ for all $t \geq 1$. Therefore, storage dominates real balances in rate of return, and, by Eqs. (2a) and (2b), implies that the reserve requirement is binding. Thus, letting $c_1^*(t)$, $c_2^*(t)$, $q^*(t)$, and $s^*(t)$ be the solution to this problem, it follows that $q^*(t) = \beta$, $s^*(t) = 1 - \beta$, $c_1^*(t) = \beta + X(1 - \beta)$, and $c_2^*(t) = \beta + X^2(1 - \beta)$. As for the members of generations -1 and 0, a quick calculation for $p(1)$ and $p(2)$ shows that $c_2^*(-1) = c_1^*(0) = c_2^*(0) = \beta$.

In contrast to the economy with private information, one can consider an alternative competitive economy in which the type of each member of generation $t \geq 1$ becomes publicly observable at time $t + 1$. As in Diamond and Dybvig (1983), this permits the members of generation t to write insurance contracts that allow for (as of time t) the optimal sharing of real balances and storage between the two types of agents. At time $t \geq$

[11] The remaining quantity of currency, $(1 - \lambda)H/(2 - \lambda)$, is held by the late diers of generation $t - 1$. Since they do not consume at time t, they withhold their stock of currency from the market.

1, the optimal contract solves the following problem: choose $c_1(t)$, $c_2(t)$, $q(t)$, and $s(t)$ to maximize $\lambda u(c_1(t)) + (1 - \lambda)u(c_2(t))$ subject to

$$\frac{\lambda c_1(t)}{R_1(t)q(t) + Xs(t)} + \frac{(1 - \lambda)c_2(t)}{R_2(t)q(t) + X^2 s(t)} = 1, \tag{4a}$$

$$q(t) + s(t) = 1, \tag{4b}$$

$$q(t) \geq \beta, \tag{4c}$$

$$c_2(t) \geq R_1(t + 1)c_1(t), \tag{4d}$$

where Eq. (4d) is a self-selection constraint on the part of late diers. Specifically, Eq. (4d) requires that late diers prefer to consume the quantity of consumption provided to them under the optimal contract than to acquire the optimal quantity of consumption provided to early diers and then convert it to real balances.

Let $\rho_1(t) = R_1(t)q(t) + Xs(t)$, the endogenously determined average one-period rate of return, and $\rho_2(t) = R_2(t)q(t) + X^2 s(t)$, the endogenously determined average two-period rate of return. Then, after substituting Eq. (4b) into (4a), the first-order conditions for this problem can be written as

$$\lambda u_1' - \phi\lambda\rho_1(t)^{-1} - \nu R_1(t + 1) = 0, \tag{5a}$$

$$(1 - \lambda)u_2' - \phi(1 - \lambda)\rho_2(t) + \nu = 0, \tag{5b}$$

$$\phi[\lambda c_1(t)\rho_1(t)^{-2}(X - R_1(t)) + (1 - \lambda)c_2(t)\rho_2(t)^{-2}(X^2 - R_2(t))] \leq \mu;$$

$$= \mu \text{ if } s(t) > 0, \tag{5c}$$

$$\mu(1 - s(t) - \beta) = 0, \tag{5d}$$

$$\nu[c_2(t) - R_1(t + 1)c_1(t)] = 0, \tag{5e}$$

where ϕ, μ, and ν are nonnegative Lagrange multipliers for Eqs. (4a), (4c), and (4d).

Let $c_1^{**}(t)$, $c_2^{**}(t)$, $q^{**}(t)$, and $s^{**}(t)$ be the solution to this problem. The assumption that agents in the public information economy behave competitively implies that Eq. (3) must hold at every date. Hence, it is again the case that $R_1(t) = R_2(t) = 1$ for all $t \geq 1$, and, by Eqs. (5c) and (5d), that the reserve requirement is binding; $q^{**}(t) = \beta$ and $s^{**}(t) = 1 - \beta$. This in turn implies that if $c_2^{**}(t) > c_1^{**}(t)$, then by Eqs. (5a), (5b), and (5e), $c_1^{**}(t)$ and $c_2^{**}(t)$ must satisfy $[\beta + X(1 - \beta)]u'(c_1^{**}(t)) = [\beta + X^2(1 - \beta)]u'(c_2^{**}(t))$. Since $X > 1$, this condition is consistent with $c_2^{**}(t) > c_1^{**}(t)$ so that late diers do in fact self-select. Furthermore, as in Diamond and Dybvig (1983), $c_1^{**}(t) > c_1^*(t)$ and $c_2^{**}(t) < c_2^*(t)$.[12] Finally, calculating $p(1)$ and $p(2)$ shows that under the optimal contract, $c_2^{**}(-1) = c_1^{**}(0) = c_2^{**}(0) = \beta$.

4. EQUILIBRIA IN THE PRESENCE OF AN INTERMEDIARY

The existence of private information implies that the competitive economy cannot support a market for Arrow–Debreu state-contingent claims. Consequently, the optimal risk-sharing contract cannot be achieved as a competitive equilibrium. Fortunately, however, as in Diamond and Dybvig (1983) and others, since the optimal contract is incentive compatible, an intermediary that offers "bank accounts" can achieve the optimal contract as a pure strategy Nash equilibrium.[13] Due to the presence of the reserve requirement, here a bank account represents a claim to fiat currency and the output of the investment technology. To distinguish between these two parts of a bank account, I use the terms *real balances* to denote the bank's holdings of currency and *deposits* to denote the bank's holdings of storage.

In order to keep the form of the bank account as simple as possible, I make the following two assumptions. First, the bank knows the generational identity of each of its depositors. Note that this assumption does not overturn the one made earlier that the bank cannot identify a given depositor's type. Second, the bank cannot use the deposits or the real balances of generations $t - 1$ and $t + 1$ to help finance the withdrawals of generation t. This second assumption, while common to much of the literature on bank runs, is not usually made explicit.[14]

At time $t \geq 1$, each young agent supplies his entire endowment to the bank which then allocates this between real balances and storage.[15] In exchange, the bank offers to pay each member of generation t who withdraws at time $t + 1$ the greater of $\rho_1(t)$, which is endogenously determined, or $r_1(t)$, a fixed one-period rate of return which is determined by the bank. The bank continues to serve agents in the random order in

[12] The proof of this result is a straightforward application of Diamond and Dybvig's risk-sharing proof. For details, see their footnote 3 and set their ρ to 1, the one-period return (1 for them) to $\beta + (1 - \beta)X$, and the two-period return (R for them) to $\beta + (1 - \beta)X^2$.

[13] Of course such an intermediary also has a second Nash equilibrium, one which corresponds to a bank run. See Section 4.2.

[14] Modeling bank runs in the absence of this assumption appears to be an interesting topic for future research.

[15] With no loss of generality, one can assume that once the bank has allocated each agent's endowment, it returns to the agent the optimal quantity of real balances. Under this interpretation, agents hold real balances while the bank holds deposits.

which they arrive until either all who seek to withdraw do so, or until the bank's assets held on behalf of generation t are exhausted. Should the bank have assets remaining, they are held until time $t + 2$ at which time they are distributed equally among the members of generation t who did not withdraw at time $t + 1$. Drawing upon Diamond and Dybvig's notation, this implies that $V_1(t)$ and $V_2(t)$, the time $t + 1$ and $t + 2$ payoffs to members of generation t, are given by

$$V_1(t, f_j) = \begin{cases} \rho_1(t) & \text{if } r_1(t) \le \rho_1(t) \\ \left.\begin{matrix} r_1(t), & f_j < r_1(t)^{-1}\rho_1(t) \\ 0, & f_j \ge r_1(t)^{-1}\rho_1(t) \end{matrix}\right\} & \text{if } r_1(t) > \rho_1(t) \end{cases} \tag{6a}$$

$$V_2(t, f) = \begin{cases} 0 & \text{if } r_1(t) \le \rho_1(t) \\ \max\left\{\dfrac{\rho_2(t)}{\rho_1(t)} \dfrac{[\rho_1(t) - r_1(t)f]}{(1 - f)}, 0\right\} & \text{if } r_1(t) > \rho_1(t), \end{cases} \tag{6b}$$

where f_j is the share of accounts from generation t that are withdrawn before agent j arrives at the bank, and f is the total share of accounts that are ultimately withdrawn by members of generation t.

4.1. *No Bank Runs*

Let the bank set $r_1(t) = c_1^{**}(t)$. Then the bank account described above yields the optimal risk-sharing contract of Section 3 as a pure strategy Nash equilibrium. To see this, note first that if bank runs never occur (i.e., $f = \lambda$), then Eq. (3) implies that $R_1(t) = R_2(t) = 1$, which in turn implies that the reserve requirement binds. Since the bank allocates β to real balances and $1 - \beta$ to storage every period, it follows that $\rho_1(t) = \beta + X(1 - \beta) \equiv \rho_1^o(t)$ and $\rho_2(t) = \beta + X^2(1 - \beta) \equiv \rho_2^o(t)$, their values under optimal risk sharing. Second, since $\rho_1^o(t) < c_2^{**}(t)$, Eqs. (4a), (6a), and (6b) imply that $V_1(t) = c_1^{**}(t)$ and $V_2(t) = c_2^{**}(t)$. Last, since $c_1^{**}(t) < c_2^{**}(t)$, late diers do not withdraw until period $t + 2$. Hence $f = \lambda$ is indeed a Nash equilibrium in which bank runs do not occur.

When $c_1^{**}(t) > \rho_1(t)$, any member of generation t who withdraws at time $t + 1$ receives more of the consumption good than his share of the bank's assets are currently worth. Since each such agent is only concerned with the magnitude of his withdrawal and not with its shares of real balances and storage, henceforth I assume that the bank pays out real balances and storage in the same proportions that it currently holds these assets, namely $R_1(t)q(t)/\rho_1(t)$ for real balances and $Xs(t)/\rho_1(t)$ for storage. Thus, in the absence of bank runs, the early diers of generation t receive $\beta c_1^{**}(t)/\rho_1^o(t)$ in real balances and $X(1 - \beta)c_1^{**}(t)/\rho_1^o(t)$ in storage.

4.2. *Bank Runs*

The deposit contract's second Nash equilibrium, namely a bank run, occurs when all members of generation t seek to withdraw their funds from the bank at time $t + 1$. To verify that a bank run is in fact an equilibrium, suppose that member j of generation t anticipates that all other members of his generation will run the bank. Since the bank pays out funds as long as it is able (or there are agents to serve), agent j obtains strictly positive expected consumption by participating in the run. Should he not participate, then his consumption is necessarily zero. Therefore, running the bank is also agent j's optimal strategy.

Like Diamond and Dybvig (1983) and Freeman (1988), I assume that at time $t + 1$, if agents observe that a commonly viewed (possibly exogenous) random event has occurred (e.g., sunspots; see Azariadis, 1981; Cass and Shell, 1983), then members of generation t run the bank.[16] Naturally, if the probability of such an event were high enough, then agents would allocate part of their endowment to the intermediary and directly invest the rest. Since pursuing this line of analysis would complicate the model, but not greatly change the results, I assume that the probability that a bank run occurs is sufficiently small that agents choose to place their entire endowment in the bank.

There exist a number of mechanisms that can prevent a bank run equilibrium from occurring and do so without altering the bank's optimal contract. Examples include suspension of payments (Diamond and Dybvig, 1983);[17] deposit insurance (Bryant, 1980; Diamond and Dybvig, 1983; Freeman, 1988); sales of bank equities (Jacklin, 1987); interbank lending using a central bank (Bhattacharya and Gale, 1987); and a correspondent banking system (Smith, 1988). Though each of these mechanisms is discussed within the context of a model in which fiat currency is absent, they can all be applied to the model presented here. However, from a historical standpoint, a case can be made that discount loans from the Federal Reserve and a suspension of payments are the only two mechanisms that are relevant to the 1929 to 1933 period. Nevertheless, the Fed did not extend significant amounts of credit to the U.S. banking system during this time, and President Franklin Roosevelt did not suspend payments until March 6, 1933, at the very end of the period under consideration.

[16] In contrast to this approach, Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), and (less formally) Bryant (1980) focus on the ability of an economic fundamental, private information concerning asset payoffs, to trigger a run. In future work, I intend to apply this type of structure to a model of currency and deposits.

[17] Recently, Engineer (1989) has shown that if the horizon in Diamond and Dybvig's (1983) economy is increased to four periods and a subset of agents who have preferences weighted toward consumption in the fourth period is added, then a suspension of payments does not always prevent a bank run.

Since the mechanisms described above either did not exist or were not employed during the period under discussion, there is no loss (historically speaking) in not applying them to the model of this paper.[18]

5. EMPIRICAL IMPLICATIONS

Suppose that no bank run has occurred through time t. Since the probability that one will occur at time $t + 1$ is (effectively) zero, at time t the bank allocates β of each young agent's endowment to real balances and $1 - \beta$ to storage. Should a run in fact occur at time $t + 1$, then by Eq. (6a), every member of generation t seeks to withdraw the greater of $\rho_1(t) = R_1(t)\beta + X(1 - \beta) \equiv \rho_1^r(t)$ or $c_1^{**}(t)$ from the bank. If $\rho_1^r(t) < c_1^{**}(t)$, then $\hat{f} = \rho_1^r(t)/c_1^{**}(t)$ is the measure of agents who successfully withdraw, each receiving $c_1^{**}(t)$, while all other agents receive 0. If $\rho_1^r(t) \geq c_1^{**}(t)$, then all members of generation t successfully withdraw $\rho_1^r(t)$. This second case, which (to the best of my knowledge) has no counterpart elsewhere in the literature, reflects the fact that with currency in the model, the rate of return on the bank's assets is endogenous. Furthermore, a deflation large enough to push $\rho_1^r(\mathrm{t})$ above $c_1^{**}(t)$ cannot be ruled out (see footnote 19). Since the bank is assumed to be unable to apply the storage or real balances of generation t toward the withdrawals of members of any other generation, it must pay out $\rho_1^r(t)$ to each member of generation t. Fortunately, as will be shown below, the model's results are robust across these two cases.

Recall that currency is assumed to have a liquidity advantage over storage. Therefore, if a bank run occurs at time $t + 1$, then each late dier of generation t who successfully withdraws will sell the share of his withdrawal in the form of storage for currency. Given that agents arrive at the bank in random order, it follows that if $c_1^{**}(t) > \rho_1^r(t)$, then the measure of late diers who withdraw equals $(1 - \lambda)\hat{f}$. In this case, market clearing during a bank run requires that

$$q(t + 1) + (1 - \lambda)\hat{f}X(1 - \beta)c_1^{**}(t)/\rho_1^r(t) = p(t + 1)H/(2 - \lambda). \quad (7)$$

After substituting for $\hat{f}$, Eq. (7) simplifies to read

$$q(t + 1) + (1 - \lambda)X(1 - \beta) = p(t + 1)H/(2 - \lambda). \quad (8)$$

On the other hand, if $c_1^{**}(t) \leq \rho_1^r(t)$, then Eq. (8) still determines market

[18] This is not to say that analyzing these mechanisms within the context of the model of this paper is uninteresting, just that it is not historically relevant.

clearing since there are now $1 - \lambda$ late diers entering the currency market, each of whom has $X(1 - \beta)$ units of storage to sell.

By definition, $p(t + 1)$ is the inverse of the time $t + 1$ price level. Since the supply of fiat currency is fixed at H, if the demand for real balances is smaller in the absence of a bank run than it is during one, then the model is consistent with FSC's observations about the behavior of the U.S. price level during the early 1930s. A comparison of Eq. (8) and Eq. (3) pushed ahead one period shows that there are two reasons why this is the case. First, the reserve requirement implies that during a run, the demand for real balances on the part of those born at time $t + 1$ must be at least equal to β, its value when no run occurs. Second, late diers represent a source of demand that only occurs during a run.

It is possible to place bounds on the size of the deflation that occurs during a bank run. Assuming that there is no run at time t (and as usual that none is expected at any future date), Eq. (3) implies that $p(t) = (2 - \lambda)\beta/H$. If there is a run at time $t + 1$, then by Eq. (8), $(2 - \lambda)[\beta + (1 - \lambda)X(1 - \beta)]/H \leq p(t + 1) \leq (2 - \lambda)[1 + (1 - \lambda)X(1 - \beta)]/H$ since $\beta \leq q(t + 1) \leq 1$. Consequently, when a run occurs, $[\beta + (1 - \lambda)X(1 - \beta)]/\beta \leq R_1(t) \leq [1 + (1 - \lambda)X(1 - \beta)]/\beta$.[19] Since the rate of inflation, $\pi(t) = R_1(t)^{-1} - 1$, it follows that

$$\frac{-(1 - \beta)[1 + (1 - \lambda)X]}{1 + (1 - \lambda)X(1 - \beta)} \leq \pi(t) \leq \frac{-(1 - \lambda)X(1 - \beta)}{\beta + (1 - \lambda)X(1 - \beta)} < 0.$$

Next, consider the deposit–currency ratio, measured here as the ratio of the bank's quantities of storage (which equals the market value of the bank's deposits) to real balances. If no run occurs at time $t + 1$, then the bank's deposits equal the sum of the storage purchased on behalf of agents born at time $t + 1$, plus the time $t + 1$ value of the storage held on behalf of the late diers of generation t. The quantity of real balances is similarly determined. Thus, in the absence of a bank run, the time $t + 1$ deposit–currency ratio equals

$$\frac{s(t + 1) + Xs(t)[1 - \lambda c_1^{**}(t)\rho_1^{o}(t)^{-1}]}{q(t + 1) + R_1(t)q(t)[1 - \lambda c_1^{**}(t)\rho_1^{o}(t)^{-1}]}. \tag{9}$$

If a bank run does occur at time $t + 1$, then all funds that the bank is

[19] This in turn implies that $\beta + (2 - \lambda)X(1 - \beta) \leq \rho_1^{r}(t) \leq 1 + (2 - \lambda)X(1 - \beta)$. However, since the coefficient of relative risk aversion must lie between 1 and ∞, it follows that $\beta + X(1 - \beta) < c_1^{**}(t) < [\beta + X(1 - \beta)][\beta + X^2(1 - \beta)]/\{\lambda[\beta + X^2(1 - \beta)] + (1 - \lambda)[\beta + X(1 - \beta)]\}$. Therefore, having $\rho_1^{r}(t) \geq c_1^{**}(t)$ is indeed a theoretical possibility. Furthermore, since the closer the coefficient of relative risk aversion is to one, the closer $c_1^{**}(t)$ is to its lower bound, the more likely it is that this situation occurs.

holding on behalf of the members of generation t are withdrawn. Supposing for the moment that only $(1 - \lambda)\hat{f}$ late diers successfully withdraw their funds, then the deposit–currency ratio becomes

$$\frac{s(t + 1)}{q(t + 1) + [R_1(t)q(t) + Xs(t)]c_1^{**}(t)(1 - \lambda)\hat{f}\rho_1^r(t)^{-1}}, \tag{10}$$

where the second term in the denominator reflects the real balances late diers received directly plus the amount that they acquire by selling their storage. Substituting for $\hat{f}$ implies that Eq. (10) simplifies to

$$\frac{s(t + 1)}{q(t + 1) + [R_1(t)q(t) + Xs(t)](1 - \lambda)}. \tag{11}$$

If every member of generation t successfully withdraws, then Eq. (11) is still valid, there being $1 - \lambda$ such agents, each of whom receives $R_1(t)q(t)$ in real balances and sells $Xs(t)$ units of storage.

When no bank run occurs, $q(t) = q(t + 1) = \beta$, $s(t) = s(t + 1) = 1 - \beta$, and $R_1(t) = 1$. Therefore, Eq. (9) $> (1 - \beta)/\beta$. On the other hand, when a run does occur, while $q(t)$ and $s(t)$ are unchanged, $q(t + 1) \geq \beta$, $s(t + 1) \leq 1 - \beta$, and $R_1(t) > 1$. Therefore, Eq. (11) $< (1 - \beta)/\beta$. These two inequalities imply that the deposit–currency ratio unambiguously falls during a bank run. This result is consistent with FSC.

Last, define the time $t + 1$ money supply, $M(t + 1)$, as the nominal value of the sum of real balances plus deposits. Therefore, $M(t + 1)$ equals the sum of the numerator and the denominator of the deposit–currency ratio divided by the value of money, $p(t + 1)$. When bank runs do not occur, $M(t + 1) = [1 + \beta + X(1 - \beta) - \lambda c_1^{**}(t)]H/(2 - \lambda)\beta \equiv M^o(t + 1)$. Substituting for $c_1^{**}(t)$ with its upper bound (see footnote 19), and recalling that $\rho_2^o(t) > \rho_1^o(t)$, implies that

$$\begin{aligned} M^o(t + 1) &> \frac{\{\beta + X^2(1 - \beta) + (1 - \lambda)[\beta + X(1 - \beta)]^2\}H}{(2 - \lambda)[\beta + X^2(1 - \beta)]\beta} \\ &\equiv M_1^o(t + 1). \end{aligned}$$

On the other hand, should a bank run occur, then $M(t + 1) = \{1 + (1 - \lambda)[R_1(t)\beta + X(1 - \beta)]\}/p(t + 1) \equiv M^r(t + 1)$. Substituting for $R_1(t)$ with its upper bound and for $p(t + 1)$ with its lower bound implies that

$$\begin{aligned} M^r(t + 1) &\leq \frac{\{1 + (1 - \lambda)[1 + (2 - \lambda)X(1 - \beta)]\}H}{(2 - \lambda)[\beta + (1 - \lambda)X(1 - \beta)]} \\ &\equiv M_u^r(t + 1). \end{aligned}$$

To establish that the money supply falls during a bank run, it suffices to show that $M_u^r(t+1) < M_l^o(t+1)$. Inspection of these two expressions shows first that if $\beta = 1$, then $M_l^o(t+1) = M_u^r(t+1)$, and second that as $\beta \to 0$, $M_u^r(t+1)$ is bounded above while $M_l^o(t+1)$ is not. Therefore, $M_u^r(t+1) < M_l^o(t+1)$ whenever β is sufficiently small. Assuming, as seems reasonable, that this restriction is satisfied, then the reduction in the money supply occurs even though the stock of high-powered money, H, does not change. Since FSC report that during the early 1930s the money supply fell by a much larger percentage than did the monetary base, the model is consistent with this observation.

6. CONCLUDING REMARKS

In order to provide a general equilibrium analysis of the macroeconomic effects of bank runs, it is necessary to have a model of financial intermediation in which there exist equilibria that correspond to both the presence and the absence of bank runs. The risk-sharing model of financial intermediation of Bryant (1980) and Diamond and Dybvig (1983) provides such an environment. However, most models in the risk-sharing tradition either include deposits, but not currency, or imply that currency and deposits are perfect substitutes, at least in the absence of bank runs. Since three of the primary effects of bank runs are the declines in the price level, deposit–currency ratio, and the money supply, most risk-sharing models are not well suited to confront these observations.

This paper offers a model of financial intermediation in the risk-sharing tradition in which there exists a type of reserve requirement on fiat currency. The reserve requirement guarantees that banks always hold currency, and that currency and deposits do not earn the same rates of return in the absence of a bank run. Therefore, the quantities of currency and deposits that a bank holds both in the presence and in the absence of a bank run can be described in a meaningful way. This in turn implies that the three variables of interest are each well defined in both the bank run and the no bank run equilibrium. A comparison of the price level, deposit–currency ratio, and money supply across the two equilibria shows that each is lower in the bank run equilibrium than in the no bank run equilibrium. In this way, the model successfully captures the three primary macroeconomic effects of bank runs under consideration.

REFERENCES

AZARIADIS, C. (1981). Self-fulfilling prophecies, *J. Econ. Theory* **25**, 380–396.

BHATTACHARYA, S., AND GALE, D. (1987). Preference shocks, liquidity, and central bank policy, *in* "New Approaches to Monetary Economics" (W. A. Barnett and K. J. Singleton, Eds.), pp. 69–88. Cambridge Univ. Press, Cambridge, UK.

BRYANT, J. (1980). A model of reserves, bank runs, and deposit insurance, *J. Banking Finance* **4,** 335–344.

BRYANT, J., AND WALLACE, N. (1980). "A Suggestion for Further Simplifying the Theory of Money," Federal Reserve Bank of Minneapolis Staff Report No. 62.

CAGAN, P. (1965). "Determinants and Effects of Changes in the Stock of Money, 1875–1960." Columbia Univ. Press, New York.

CASS, D., AND SHELL, K. (1983). Do sunspots matter? *J. Polit. Econ.* **91,** 193–227.

CHARI, V. V., AND JAGANNATHAN, R. (1988). Banking panics, information, and rational expectations equilibrium, *J. Finance* **43,** 749–763.

DIAMOND, D. W., AND DYBVIG, P. H. (1983). Bank runs, deposit insurance, and liquidity, *J. Polit. Econ.* **91,** 401–419.

ENGINEER, M. (1989). Bank runs and the suspension of deposit convertibility, *J. Monet. Econ.* **24,** 443–454.

FREEMAN, S. (1988). Banking as the provision of liquidity, *J. Bus.* **61,** 45–64.

FRIEDMAN, M., AND SCHWARTZ, A. J. (1963). "A Monetary History of the United States, 1867–1960." Princeton Univ. Press, Princeton, NJ.

HICKS, J. R. (1935). A suggestion for simplifying the theory of money, *Economica N.S.* **2,** 1–19.

JACKLIN, C. J. (1987). Demand deposits, trading restrictions, and risk sharing, *in* "Contractual Arrangements for Intertemporal Trade" (E. C. Prescott and N. Wallace, Eds.), pp. 26–47. Univ. of Minnesota Press, Minneapolis.

JACKLIN, C. J., AND BHATTACHARYA, S. (1988). Distinguishing panics and information-based bank runs: Welfare and policy implications, *J. Polit. Econ.* **96,** 568–592.

KEYNES, J. M. (1930). "A Treatise on Money," Vol. I, "The Pure Theory of Money." Macmillan & Co., London.

MILLER, P. J., AND WALLACE, N. (1985). International coordination of macroeconomic policies: A welfare analysis, *Fed. Res. Bank Minn. Rev.* **9** (Spring), 14–32.

PATINKIN, D. (1987). Real balances, *in* "The New Palgrave: A Dictionary of Economics" (J. Eatwell, M. Milgate, and P. Newman, Eds.), Vol. 4, pp. 98–101. Stockton Press, New York.

SMITH, B. D. (1988). "Bank Panics, Suspensions, and Geography: Some Notes on the 'Contagion of Fear' in Banking," MS, University of Western Ontario.

WALDO, D. G. (1985). Bank runs, the deposit–currency ratio and and the interest rate, *J. Monet. Econ.* **15,** 269–277.

WALLACE, N. (1984). Some of the choices for monetary policy, *Fed. Res. Bank Minn. Rev.* **8** (Winter), 15–24.